<u>Mathematical Practices in Pearson's *Prentice Hall Algebra 1, Geometry, Algebra 2*</u>

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

The structure of the Pearson's *Prentice-Hall High School Mathematics Program* supports students in making sense of problems and in persevering in solving them. With the *Solve It!,* a problem situation that opens each lesson, students look to uncover the meaning of the problem and propose solution pathways. In the Teacher's Edition are guiding questions that teachers can ask to help students persevere in finding a workable entry point for the problem. The rich visual support to many of the Problems helps students make sense of the context in which the problems are set. Students realize the importance of checking their solution plans and answers through the prompts in the *Think, Plan,* and *Know-Need-Plan* boxes. For example, in a *Think* box students will be asked, "How can you get started?"; in a *Plan* box "How is this inequality different from others you've solved?" With the *Think About a Plan* Exercises, students analyze the givens in a problem situation and then formulate a solution plan. Each lesson also has a set of Challenge Exercises in which students consider previously-solved problems and persevere to formulate a solution plan. In the *Pull It All Together* activity at the end of each chapter, students apply their sense-making and perseverance skills to solve real world problems. As students progress through the program, the scaffolding becomes less structured and students analyze a problem situation and formulate solutions plans with greater autonomy and proficiency.

For examples in Algebra 1, see pages 48, 53, 104, 208, 321, 588
For examples in Geometry, see pages 69, 159, 205, 266, 456, 479
For examples in Algebra 2, see pages 39, 182, 250, 346, 440, 486, 742

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the 2 units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Reasoning is one of the guiding principles of the Pearson's *Prentice Hall High School Mathematics Program* and is especially evident in the *Think*, *Plan*, *Write*, and *Know-Need-Plan* boxes through which students are guided to represent problem situations symbolically. These features offer students prompts to facilitate the development of abstract and quantitative reasoning. Students regularly represent problem situations symbolically as they express the problem using algebraic and numeric expressions. Through the solving process, as they manipulate expressions, students are reminded to check back to the problem situation to verify the referents for the expressions. Each lesson ends with a *Do You UNDERSTAND?* feature in which students explain their thinking related to the concepts studied in the lesson. Throughout the Exercise sets are Reasoning Exercises that focus students' attention on the structure or meaning of an operation rather than the solution. For example, students are asked to determine the numbers in a subset without listing each subset. For the *Pull It All Together* feature at the end of each chapter, students draw on their reasoning skills to put forth an accurate symbolic representation of the problem presented and to formulate and execute a logical solution plan.

For examples in Algebra 1, see pages 47, 48, 111, 201, 262, 334, 457, 549, 702, 730
For examples in Geometry, see pages 62, 115, 219, 266, 335, 355, 473, 511, 618, 728, 785
For examples in Algebra 2, see pages 20, 116, 139, 230, 300, 438, 511, 574, 657, 675, 721, 767

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They *make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.*

Consistent with a focus on reasoning and sense making is a focus on critical reasoning – argumentation and critique of arguments. In Pearson's *Prentice Hall High School Program,* students are frequently asked to explain their solutions and the thinking that led them to these solutions. The many *Reasoning* Exercises found throughout the program specifically call for students to formulate arguments to support their solutions. In the *Compare and Contrast* Exercises, students are also expected to advance arguments to explain similarities or differences or to weigh the appropriateness of different strategies. The Error Analysis Exercises found in each lesson require students to analyze and critique the solution presented to a problem.

For examples in Algebra 1, see pages 109, 172, 244, 339, 369, 501, 760
For examples in Geometry, see pages 59, 154, 253, 319, 455, 510, 649, 733
For examples in Algebra 2, see pages 26, 92, 205, 240, 329, 434, 507, 622, 769, 853

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's *Prentice Hall High School Mathematics Program* build mathematical models using functions, equations, graphs, tables, and technology. For each *Solve It!* and *Pull It All Together* activities, students apply a mathematical model to the real-world problem presented. Students' skills applying mathematical models to problem situations are refined and honed through the prompts offered in the *Think, Plan,* and *Know-Need-Plan* boxes. These prompts become less structured as students advance through the program and become more skilled at modeling with mathematics.

For examples in Algebra 1, see pages 12, 33, 62, 126, 255, 370, 522, 543, 581, 588, 600
For examples in Geometry, see pages 60, 174, 356, 432, 517, 665, 727, 804
For examples in Algebra 2, see pages 94, 134, 210, 334, 471, 544, 582, 706, 839, 883, 923

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical devices to technological tools. They use various manipulatives in *Activity Concept Bytes* and different technology tools in the *Technology Concept Bytes*. By developing fluency in the use of different tools, students are able to select the appropriate tool(s) to solve a given problem. The *Choose a Method* Exercises strengthen students' ability to articulate the difference in use of various tools. Technology and technology tools, such as graphing calculators, dynamic math tools, and spreadsheets, are an integral part of Pearson's *Prentice Hall High School Mathematics Program* and is used in these ways:

- to develop understanding of mathematical concepts;
- to solve problems that would be unapproachable without the use of technology; and

- to build models based on real-world data.

For examples in Algebra 1, see pages 59, 101, 260, 336, 487, 536, 763
For examples in Geometry, see pages 49, 147, 225, 300, 352, 470, 515, 659, 741, 789
For examples in Algebra 2, see pages 163, 215, 318, 413, 459, 594, 621, 772, 835, 927

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms are highlighted in each lesson and Key Concepts explained in the Take Note features. In the *Do You UNDERSTAND?* feature, students revisit these key terms and provide explicit definitions or explanations of the terms. For the *Writing* Exercises, students are once again expected to provide clear, concise explanations of terms, concepts, or processes or to use specific terminology accurately and precisely. Students are reminded to use appropriate units of measure when working through solutions and accurate labels on axes when making graphs to represent solutions.

For examples in Algebra 1, see pages 20, 37, 202, 268, 294, 384, 451, 494, 565, 741
For examples in Geometry, see pages 17, 92, 175, 253, 378, 510, 629, 689, 784
For examples in Algebra 2, see pages 22, 78, 145, 229, 300, 372, 462, 519, 622, 691, 777, 875

7. Look for and make use of structure.

*Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 *8 equals the well-remembered 7 *5 +7 *3, in preparation for learning about the distributive property. In the expression* $x^2 + 9x + 14$*, older students can see the 14 as 2 *7 and the 9 as 2 + 7.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see* $5 - 3(x\ y)^2$ *as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers* x *and* y*.*

Throughout the program, students are encouraged to discern patterns and structure as they look to formulate solution pathways. Through the *Think, Plan,* and *Know–Need–Plan* boxes, students are prompted to look within a problem situation and seek to break down the problem into simpler problems. This is especially encouraged in *Geometry,* where students think about composing and decomposing two–dimensional or three–dimensional figures to uncover a structure from which generalizable statements can be formulated. The *Pattern*/*Look for a Pattern* Exercises explicitly ask students to find patterns in operations or graphic displays.

For examples in Algebra 1, see pages 28, 55, 57, 65, 183, 242, 303, 418, 670
For examples in Geometry, see pages 60, 141, 230, 313, 387, 461, 557, 623, 700, 804
For examples in Algebra 2, see pages 6, 68, 173, 217, 327, 375, 442, 509, 583, 641, 674, 776, 838, 913

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, through the *Think, Plan,* and *Know–Need–Plan* boxes, students are prompted to look for repetition in calculations to devise general methods or shortcuts that can make the problem–solving process more efficient. Students are prompted to look for similar problems they have previously encountered or to generalize results to other problem situations. The Dynamic Activities, a feature at PowerAlgebra.com and PowerGeometry.com, offer students opportunities to notice regularity in the way operations or functions behave by easily inputting different values.

For examples in Algebra 1, see pages 55, 109, 189, 242, 294, 376, 489, 570
For examples in Geometry, see pages 83, 175, 265, 295, 371, 451, 499, 652, 726, 782
For examples in Algebra 2, see pages 108, 135, 220, 328, 383, 464, 565, 624, 839